Title: Algebraic Geometry.

The successful candidate will work in the scope of the PRIN project "Symplectic varieties: their interplay with Fano manifolds and derived categories", in the broad area of Algebraic Geometry. Depending on the skills of the best candidate, they will be given one of the following problems, to be solved in collaborations with members of the group, specifically Bologna's unit:

H2: Study finite maps of HK manifolds and to produce maximal projective families of them.

By results of Bayer, Macrì, Matsushita, Markman, Yoshioka, Rapagnetta, Onorati and Mongardi, known HK manifolds with a nef square zero divisor have a lagrangian fibration with base P^n. It follows that finding a manifold with a finite map to $P^n \times P^n$ is equivalent to finding a manifold with two nef classes of square zero. The easiest way to do so is to take a Picard rank two manifold with nef cone which coincides with the closure of the positive cone. One can take a manifold without any wall divisors in its Picard lattice: a good rank two candidate is given by lattices of the form $U(r)$, where U is the hyperbolic lattice and $r>1$. If X is of $K3^{[n]}$ type and $n=r=2$, this is the case studied by Iliev, Kapustka, Kapustka and Ranestad. For the remaining n and r, all these manifolds are moduli spaces of twisted sheaves on $K3$ surfaces by a criterion of Huybrechts, hence they can be constructed and their maps to $P^n \times P^n$ can be studied. After enough examples are obtained, it is possible to start classifying these covers, by understanding the possible ramifications. To construct new maximal projective families the project will start by constructing degenerate examples, that is pairs (X,H) where X is of the desired deformation type and H has the desired degree but it is not ample and only nef. Such examples will be given by moduli spaces and two different degenerate behaviours could happen. Firstly, the map induced by H could have a degree higher than the general element of the projective family. Secondly, the map could contract a subvariety of X. While little can be done in the first case, the second case can give more insight on the general family, as the image of the map is a mild degeneration of the image of the corresponding general case. The first cases are pairs (X,H) where X is of $K3^{[n]}$ type and H has square 2. The map has conjecturally degree two by O'Grady, and this is the case if $n<5$. The image of the map has the expected codimension of the n-th degeneracy locus of a map between sheaves. Indeed, if $n=1,2$, the image of the map is constructed as a degeneracy locus and, if $n=3,4$ O'Grady constructs some special cases of this family as a component of a degeneracy locus. One of the aims of the project is to understand the general behaviour of these families, starting from $n=3$, where a further degeneration can be constructed by taking an element of the family constructed by Iliev, G. Kapustka, M. Kapustka and Ranestad where the lagrangian in $\Wedge^3 C^6$ contains a decomposable element corresponding to a three space W in C^6. In this case, the sixfold they construct is singular along a divisor and the projection of it from the tangent in $Gr(3,6)$ to W lands in P^9.

H4: Prove special cases of the monodromy conjecture on the Motivic Zeta function for HK manifolds. The aim is to investigate the interplay between the arithmetic of a K-trivial variety defined over the field of complex Laurent series, and the geometry of its degenerations. For any such variety one can define an invariant called the motivic zeta function. This is a formal power series in one variable with coefficients in a Grothendieck ring of varieties, which encodes a wealth of information about degenerations of the variety. Its coefficients, also called motivic volumes,

measure the size of the set of rational points over ramified extensions of the ground field. The motivic zeta function is known to be rational, but its finer properties are poorly understood, as the nature of its poles. The most important open question is the monodromy conjecture, which predicts that any pole gives rise to a monodromy eigenvalue on the cohomology. The monodromy conjecture is still wide open in general, but it has been proved for abelian varieties, and several classes of K3 surfaces. Very recent progress has been made by Pagano (in his PhD thesis, supervised by Halle), who proved that if the monodromy conjecture holds for a K3 surface, then it also holds for any Hilbert scheme of points on the surface. This project seeks to investigate the relationship between the respective zeta functions of a K3 surface S and a (HK) moduli space of sheaves on S. Like in the Hilbert scheme case, where any pole arises as a sum of poles for the underlying surface, we expect a close relationship between the poles of the two zeta functions. In part, this expectation is guided by the description of the cohomology ring of the moduli space as a Galois representation by Charles and Frei. Our main goal is to compute the zeta function of the moduli space in terms of the zeta function of the surface, and to extend Pagano's result to more general moduli spaces. An intermediate goal is to refine the methods used for Hilbert schemes, and construct an explicit weak Néron model of the moduli space. This is a certain smooth scheme over the ring of formal power series, whose special fibre determines the degree one coefficient of the zeta function. We expect that work of Inaba on moduli spaces of sheaves on a normal crossing union of surfaces will be highly useful for this endeavour. We will then identify the correct locus inside Inaba's moduli space, to make the resulting degeneration into a weak Néron model. Emphasis will be put first on explicitly understanding low dimensional moduli spaces. We stress that, beyond Hilbert schemes, little is known about motivic volumes of HK varieties in dimension higher than two.

F2: Fano Varieties of K3 type. Until recently, only a few examples of F-K3 varieties were known: cubic fourfolds, some linear and quadratic sections of Grassmannians and few more examples. The goal of this part of the project is twofold: on the one hand, our first objective is to produce more examples of such varieties. The team is already working on a more efficient computation of cohomology for subvarieties of homogeneous varieties, in order to make the methods in the literature suitable for a more extensive search in a broader setting. The first relevant objective is a complete classification of all Fano 4-folds obtained as above in products of flag varieties. The next aim would be the implementation of an exhaustive search of Fano zero loci of sections of homogeneous vector bundles over more general homogeneous varieties, providing partial classifications of Fano varieties in this setting. Relevant generalizations of this framework are extending the search to wider classes of varieties, such as degeneracy loci of morphisms between vector bundles over homogeneous varieties, or more generally orbital degeneracy loci, a wide class of varieties which has been recently introduced and which naturally generalise the notion of degeneracy loci of morphisms. On the other hand, to every example of Fano variety of K3 type we aim to explicitly associate a family of HK varieties: as a moduli space of objects in the K3 category, if we can produce stability conditions in it, or with explicit geometry otherwise, like in the case of the families of Intermediate Jacobians on cubic fourfolds. Moreover, in the case of F-K3 fourfolds an interesting open problem is also given by their rationality and stable rationality, which can often be formulated as problems of algebraicity on the associated HK varieties. For instance, the (conjectural) rationality of a cubic fourfold is related to the birational geometry of its variety of lines F, and its stable rationality is related to algebraicity of a codimension two class in FxF.

Composizione dei membri della commissione dell'eventuale Bando

La valutazione comparativa dei candidati sarà effettuata da una Commissione giudicatrice formata da: -
Prof. Giovanni Mongardi (UNIBO) (Presidente)

Dott. Enrico Fatighenti (UNIBO) (segretario verbalizzante)

Dott.ssa. Camilla Felisetti (UNIMORE) (componente)

Dott. Francesco Meazzini (UNIBO) (eventuale membro supplente).

Requisiti di ammissione

Alle selezioni sono ammessi a partecipare i candidati, anche cittadini di Paesi non appartenenti alla Unione Europea, in possesso di adeguato curriculum scientifico professionale e di:

- Dottorato o titolo equivalente in Matematica, Fisica o settori affini con adeguato curriculum scientifico-professionale.

- è previsto un colloquio;

- in caso di colloquio indicare la modalità: mista;

- non è prevista una valutazione di competenza della lingua inglese;